DISPERSION IN THE COAXIAL CABLES #4 WJW 12/31/90

The coaxial cables that we use for signal transmission everywhere exhibit some dispersion. It amounts to a few degrees of phase shift in excess of what an ideal cable would have. For most of the cables this nonlinear frequency dependent phase shift is indistinguisnable from what is produced in the various other components, and we remove it in our normal bandpass calibration. However, it is more of a nuisance in the variable delay line, because the delay steps are changed frequently during the course of an integration and the associated phase steps produce fringe phase errors which are not removed by the current calibration process. An understanding of the source of this dispersion should allow us to remove it in the correction table look-up that we are planning for the delay lines.

The magnitude of the effect is readily apparent in a set of measurements that Jim, Mark, and I made on the longest bit in one of the new delay lines. Just before the final cut, it has a delay of about 535 nsec. The table below shows measured delays at 90 Mhz, 980 MHz, and averaged over 50-1000 MHz.

f(MHz)	Delay(nsec)
90	535.820 +/- .06
980	535.341 +/- .06
50-1000	535.429 +/- .0004

One delay step in the new system is .03 nsec, so the above differences are significnt.

Most of both the dispersion and the losses in the cables is due to the skin effect in the inner and outer conductors. The resistive part of the skin effect produces the losses, and the inductive part produces the dispersion.

I. LOSSES

The skin impedance is (1+i) R_s, where $R_s = 2.61\text{x}10^{-7}f^{0.5}$ ohms for copper and $3.26\text{x}10^{-7}f^{0.5}$ ohms for aluminum. The resistance per unit length of a cylindrical conductor is R_s/(circumference) ohms/m. The Cablewave 1/2" coax has a copper inner conductor of .168" diameter and an aluminum outer conductor of .460" inner diameter, producing a total resistance of $2.88\text{x}10^{-5}f^{0.5}$ ohms/m. The attenuation coefficient is $R/2Z_0$, which is $2.50\text{x}10^{-6}f^{0.5}$ db/m for the 50 ohm line. This formula predicts an attenuation of .050 db/m at 400 MHz for the cable whereas we measure a loss of 0.063 db/m, very close to that shown in the manufacturer's data. This 26% excess may be due to impurities in the metals. The predicted $f^{0.5}$ frequency dependence is followed closely by the data up to 1000 MHz. This suggests that the effective resistance is $3.80\text{x}10^{-5}f^{0.5}$ ohms/m.

Cables with smaller diameter conductors have proportionaly higher losses. The High loss cable that we use to balance the losses of the 1/2" coax in the delay line has a smaller diameter and, more important, a nickel plated inner conductor.

II. DISPERSION

The skin effect inductance has the same magnitude as the resistive part. $2\pi fL = R = 3.80\text{x}10^{-5}f^{0.5}$, and $L = 6.05\text{x}10^{-6}f^{-0.5}$ H/m. This must be added to the normal inductance/(length), which is $(\mu_0/2\pi)(Z_0 n/60) = 2.5\text{x}10^{-7}$ H/m. (n is the index of refraction in the cable, 1.228.) Thus, the total inductance/m is $[2.05\text{x}10^{-7} + 6.05\text{x}10^{-6}f^{-0.5}]$ H/m.

The wave number is: $\beta = \omega[LC]^{0.5} \approx \beta_0[1+14.8f^{-0.5}]$, where β_0 is the β in the absence of the skin effect.

The phase is $\Phi = 360^0 f\tau_0[1+14.8f^{-0.5}]$, where τ_0 is the delay in the absence of the skin effect.

The delay is $\frac{\partial(\Phi/360)}{\partial f} = \tau_0[1+7.4f^{-0.5}]$

The table below compares the predicted and measured delays for the 535 nsec piece of cable. τ_0 is taken to be 535.200 nsec.

f(MHz)	τ_{meas}	−535.200	$[535(7.4f^{-0.5})]$
125	535.676+/−0.03	.476	.354
274	535.471+/−0.03	.271	.239
424	535.401+/−0.03	.201	.192
574	535.367+/−0.03	.167	.165
720	535.362+/−0.03	.162	.148

The formula predicts the correct order of the dispersion, but the observed amount appears to be about 10%-20% greater. On the other hand, the differences are mostly within the measurement uncertainties.

The high loss cable is cut to length to balance the <u>losses</u> of the 1/2" cable. The above analysis suggests that because the losses and dispersion are related, having a common origin in the skin effect, the short piece of high loss cable should have about the same dispersion as its fellow piece of longer low loss cable. The graph below shows the delay of a piece of the high loss cable that is actually 20% longer than the length required to balance the losses of the 535 nsec piece. The encircled dots are calculated from the correction term $[(535)(5.9f^{-0.5})]$ with an arbitrary offset. The factor 5.9 was chosen to make the best fit. Again the order of the dispersion is well predicted by the formula, but the choice of the coefficient shows that the high loss cable has about 30% less dispersion than that of the piece of low loss cable that it is paired with. The frequency dependence is well fit by the formula. Also, there will be substantial cancelation of the efect between the high loss and low loss cables. The residual difference for this pair is about 4° maximum, and the correction for that can be made accurately in the table look-up, based on the simple $f^{-0.5}$ frequency dependence.