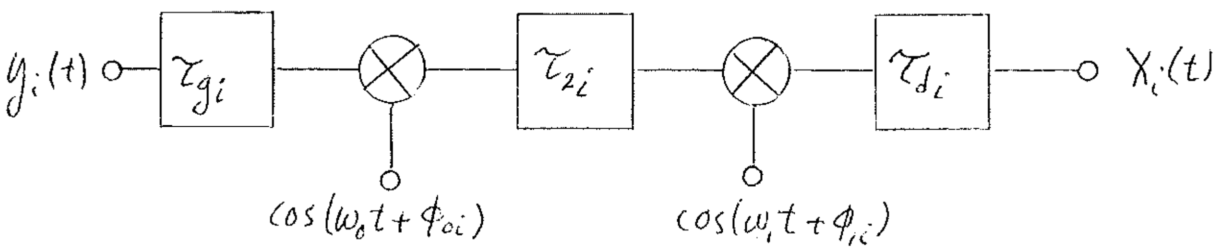
Bima memo # 36

1 LO, Delay, and Correlator Control for the BIMA Array

The RF circuit on each antenna consists of a double conversion super- heterodyne receiver with first and second local oscillators, amplifiers, and filters which define the pass-bands. Both side-bands of the first local oscillator are detected. The natural fringe rates are rotated to zero by offsetting fo the local oscillators, and the sidebands are separated by phase switching of the first local oscillator through $\pi/4$. The second local oscillator is switched by $\pi/2$ to eliminate DC drifts. The switching of both oscillators employs the orthogonal Walsh Function set in order to minimize cross-talk between the different channels. At the correlator output, the correlation functions for the various spectral windows of the IF are stored in pairs of registers in synchronism with the Walsh Functions. A complex, linear combination of each of these pairs transforms (by Fourier Transformation) into complex visibility spectra.

The diagram below shows the RF circuit, the first IF circuit, and the second IF circuit, each separated by mixers. Also shown are the upper and lower sideband positions with respect to the first LO, ν_0.



The various frequencies are defined as follows: $\omega_0 = 2\pi\nu_0$, $\omega_1 = 2\pi\nu_1$, $\nu \sim 80 - 115GHz$ or $210 - 270GHz$, $\nu_1 = 1270MHz$, ω_{if} =first IF, ω_{iif} =second IF, $\omega_u - \omega_0 = \omega_{if}$, $\omega_0 - \omega_L = \omega_{if}$, $\omega_{if} - \omega_1 = \omega_{iif}$. The input to receiver i is $y_i(t)$, and the output at the second IF for receiver i is $x_i(t)$. Both sidebands of the first LO are received.

$$y_i(t) = \int_0^\infty A_i^u(\nu)\cos[\omega_u t + \phi_i^u(\nu)]d\nu + \int_0^\infty A_i^L(\nu)\cos[\omega_L t + \phi_i^L(\nu)]d\nu \tag{1}$$

$$\phi_{0i}(t) = \int_0^t \omega_{0i}(x)dx + C_{0i} + \psi_{0i}(t) \tag{2}$$

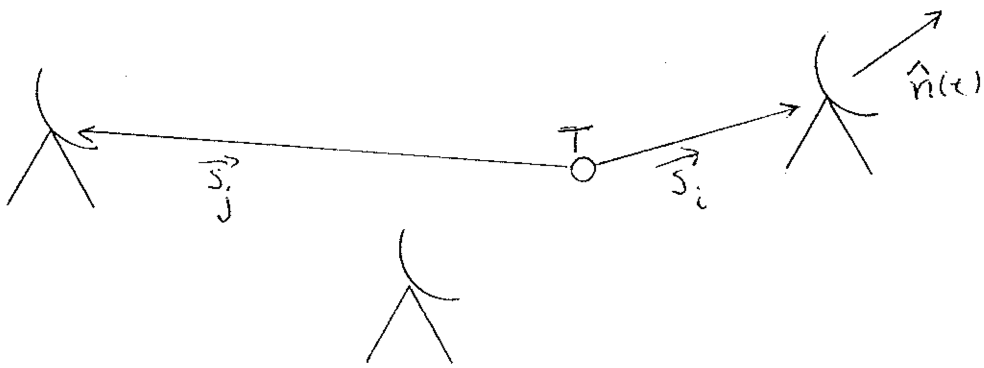
$$\phi_{1i}(t) = \int_0^t \omega_{1i}(x)dx + C_{1i} + \psi_{1i}(t) \tag{3}$$

The first phase term is the offset frequency, the second is the reset phase constant, and the third is the Walsh function switching phase. The entire first LO term is $\cos[\omega_0 t + \phi_{0i}]$, and the second LO term is $\cos[\omega_1 t + \phi_{1i}]$. The result of the multiplication at each mixer is a difference phase term and a sum phase term. The sum term is filtered out. Taking just the phase difference term at each mixer, we find:

$$x_i(t) = \int_0^\infty A_i^u \cos[\psi_i^u + \omega_{iif}t - \omega_{iif}(\tau_{gi} + \tau_{di}) - \omega_0\tau_{gi} - \omega_1\tau_{gi} - \omega_{if}\tau_{2i} - \phi_{0i} - \phi_{1i}]d\nu \quad (4)$$

$$+ \int_0^\infty A_i^L \cos[-\psi_i^L + \omega_{iif}t - \omega_{iif}t(\tau_{gi} + \tau_{di}) + \omega_0\tau_{gi} - \omega_1\tau_{gi} - \omega_{if}\tau_{2i} + \phi_{0i} - \phi_{1i}]d\nu \quad (5)$$

The next step is to adjust the delay line and the oscillator offset frequencies in order to eliminate the various phase terms in (4) and (5). Tracking the delay line, τ_{di}, eliminates phase errors over the extent of the IF band. Continuously adjusting the oscillator phases corrects the phases over the IF band at the RF frequencies. The delay and fringe rate rate are calculated with respect to a fixed point between the antennas. The reference is the T intersection. $n\hat{(t)}$ is a unit vector in the direction of the source.



The geometrical delay relative to the reference point is $\tau_{ig} = -\vec{S}_i.\hat{n}/c$. τ_m is the maximum delay in the variable delay line; $\tau_m/2$ is the midpoint. We set the delay line so that:

$$\tau_{di} = \tau_m/2 + \vec{S}_i.\hat{n}/c. \quad (6)$$

$$\tau_{di} + \tau_{gi} = \tau_m/2 \quad (7)$$

Now for each antenna, we make the fringe rate zero with respect to the reference point, the T intersection.

$$\omega_0\tau_{gi} + \int_0^t \omega_{oi}(x)dx + C_{oi} = 0 \quad (8)$$

$$\omega_1\tau_{gi} + \int_0^t \omega_{1i}(x)dx + C_{1i} = 0 \quad (9)$$

With $\tau_{gi} = -\vec{S}_i.\hat{n}/c$, the first and second local oscillator offset frequencies must be set as follows.

$$\omega_{0i} = \omega_0\frac{d}{dt}[\vec{S}_i.\hat{n}/c] \quad (10)$$

$$\omega_{1i} = \omega_1 \frac{d}{dt}[\vec{S}_i.\hat{n}/c] \tag{11}$$

The constants C_{0i} and C_{1i} are selected to make the fringe phase zero at the reset time, $t = 0$.

$$C_{0i} = \omega_0 \vec{S}_i.\hat{n}/c \tag{12}$$

$$C_{1i} = \omega_1 \vec{S}_i.\hat{n}/c \tag{13}$$

Now we assume that the τ_{2i} are all identical. If the above delay and phase corrections are made with no errors, then the output simplifies to the following expression.

$$x_i(t) = \int_0^\infty A_i^u \cos[\psi_i^u + \omega_{iif}t - \omega_{iif}(\tau_m/2) - \psi_{0i} - \psi_{1i}]d\nu \tag{14}$$

$$+ \int_0^\infty A_i^L \cos[-\psi_i^L + \omega_{iif}t - \omega_{iif}(\tau_m/2) + \psi_{0i} - \psi_{1i}]d\nu \tag{15}$$

Each output signal pair is cross-correlated for each IF window.

$$C_{ji}(\tau) = \int_{-\infty}^\infty x_i(t)x_j(t+\tau)dt \tag{16}$$

$$= \int_0^\infty A_i^u \int_0^\infty A_j^u \int_{-\infty}^\infty \cos[\Phi_1]\cos[\Phi_2]dtd\nu d\eta, \tag{17}$$

+ lower sideband products.

where $\Phi_1 = [\psi_i^u + \omega_{iif}t - \omega_{iif}\tau_m/2 - \psi_{0i} - \psi_{1i}]$ and $\Phi_2 = [\psi_j^u + \omega_{iif}(t+\tau) - \omega_{iif}\tau_m/2 - \psi_{0j} - \psi_{1j}]$. Then $\cos\Phi_1 \cos\Phi_2 = \cos[\Phi_1 - \Phi_2] + \cos[\Phi_1 + \Phi_2]$. The sum phase term is filtered out. The difference phase term has the form:

$$\int_{-\infty}^\infty \cos[(\nu - \eta)t + a]dt = [\cos a]\delta(\nu - \eta) \tag{18}$$

Thus, the innermost integral of (17) is a delta function, and that makes one of the two remaining integrals easy to evaluate. The result is:

$$C_{ji}(\tau) = \int_0^\infty U_{ji}(\nu)\cos[\Delta\psi_{ji}^u + \omega_{iif}\tau - \psi_{0j} + \psi_{0i} - \psi_{ij} + \psi_{1i}]d\nu \tag{19}$$

$$+ \int_0^\infty L_{ji}(\nu)\cos[-\Delta\psi_{ji}^L + \omega_{iif}\tau + \psi_{0j} - \psi_{0i} - \psi_{1j} + \psi_{1i}]d\nu, \tag{20}$$

where $U_{ji}(\nu) = A_i^u(\nu)A_j^u(\nu)$, $L_{ji}(\nu) = A_i^L A_j^L$, $\Delta\psi_{ji}^u = \psi_j^u - \psi_i^u$, and$\Delta\psi_{ji}^L = \psi_j^L - \psi_i^L$. The Fourier transform relation between the correlation function and the lower and upper sideband spectra is evident. In addition, there remain the phase terms resulting from the Walsh function phase switching. We will now see that the proper synchronous detection of the Walsh functions allows the separation of the sidebands. The phase switching terms are slowly varying in time and can be brought outside the integrals in (19) and (20) by means of trigonometric identities.

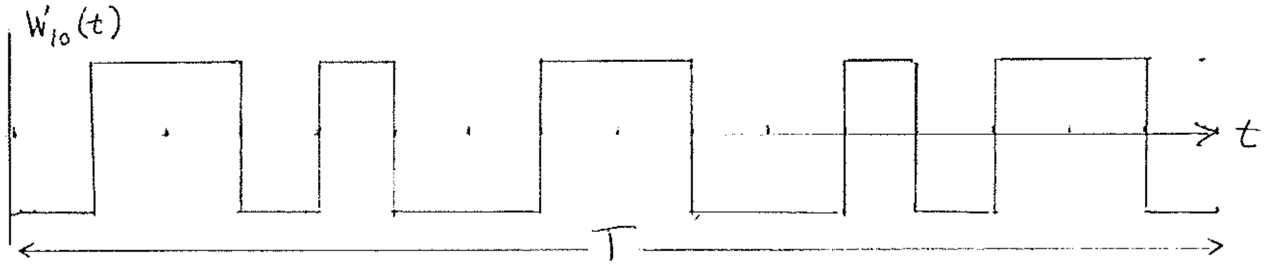
$$C_{ji}(\tau) = \cos[-\psi_{0j} + \psi_{0i} - \psi_{1j} + \psi_{1i}] \int_0^\infty U_{ji} \cos[\omega_{iif}\tau + \Delta\psi_{ji}^{u}]d\nu \tag{21}$$

$$- \sin[-\psi_{0j} + \psi_{0i} - \psi_{1j} + \psi_{1i}] \int_0^\infty U_{ji} \sin[\omega_{iif}\tau + \Delta\psi_{ji}^{U}]d\nu \tag{22}$$

$$+ \cos[\psi_{0j} - \psi_{0i} - \psi_{1j} + \psi_{1i}] \int_0^\infty L_{ji} \cos[\omega_{iif}\tau - \Delta\psi_{ji}^{L}]d\nu \tag{23}$$

$$- \sin[\psi_{0j} - \psi_{0i} - \psi_{1j} + \psi_{1i}] \int_0^\infty L_{ji} \sin[\omega_{iif}\tau - \Delta\psi_{ji}^{L}]d\nu. \tag{24}$$

The first Local Oscillator is switched by $\pi/2$ in a Walsh Function pattern, and the second Local Oscillator is switched through π by another such function:$\psi_{0i} = (\pi/4)W_{0i}(t)$, and$\psi_{1i} = (\pi/2)W_{1i}(t)$. The Walsh Functions are a digital orthogonal set. $W_n(t) = \pm 1$, and $\int_0^T Wn(t)Wm(t)dt = N_n\delta_{nm}$. The orthogonal time interval is 0.320 sec, and all integrations of the data are in multiples of this interval. Below is an example of one of the functions.



The phase terms simplify as follows. $\cos[(\pi/4)W_n(t)] = 1/\sqrt{2}$, $\sin[(\pi/4)W_n(t)] = W_n(t)/\sqrt{2}$, $\cos[(\pi/2)W_n(t)] = 0$, and $\sin[(\pi/2)W_n(t)] = W_n(t)$. With the use of these relations and some trigonometric identities, we find that the sines and cosines of the sums of the phase switching terms become:

$$\cos[-\psi_{0j} + \psi_{0i} - \psi_{1j} + \psi_{1i}] = [W_{1j}W_{1i} + W_{1j}W_{1i}W_{0i}W_{0j}] \tag{25}$$

$$- \sin[-\psi_{0j} + \psi_{0i} - \psi_{1j} + \psi_{1i}] = +[W_{0j}W_{1j}W_{1i} - W_{0i}W_{1j}W_{1i}] \tag{26}$$

$$\cos[\psi_{0j} - \psi_{0i} - \psi_{1j} + \psi_{1i}] = [W_{1j}W_{1i} + W_{1j}W_{1i}W_{0i}W_{0j}] \tag{27}$$

$$- \sin[\psi_{0j} - \psi_{0i} - \psi_{1j} + \psi_{1i}] = -[W_{0j}W_{1j}W_{1i} - W_{0i}W_{1j}W_{1i}] \tag{28}$$

These are products and sums of Walsh functions. Products of Walsh Functions are other Walsh functions; sums of Walsh functions are not. However, because they are linear combinations, the combinations are orthogonal to one another. Note that there are only two independent combinations. For brevity, we give these other names. $[W_{1j}W_{1i} + W_{1j}W_{1i}W_{0i}W_{0j}] = W^A_{ji}$, and $[W_{0j}W_{1j}W_{1i} - W_{0i}W_{1j}W_{1i}] = W^B_{ji}$. These two functions are orthogonal to each other, so long as the individual Walsh functions are properly chosen. In the spectral line correlator,the signals are passed into a third SSB mixer. For the upper sideband of this mixer, $\omega_{iif} - \omega_3 = \omega$. For the lower sideband of this mixer,$\omega_3 - \omega_{iif} = \omega$, and the spectrum is inverted. Correlations for both $\tau \leq 0$ and $\tau \geq 0$ are obtained by the interchange of the signals. The parts of the correlation function corresponding to the upper and lower sidebands can be separately identified.

$$C^u_{ji}(\tau) = \int_o^\infty U_{ji}\cos[2\pi\nu\tau + \Delta\psi^u_{ji}]d\nu \tag{29}$$

$$C^L_{ji}(\tau) = \int_0^\infty L_{ji}\cos[2\pi\nu\tau - \Delta\psi^L_{ji}]d\nu \tag{30}$$

The Fourier Transforms of these two correlation functions are the desired complex power spectra or visibility spectra for the upper and lower sidebands of the first local oscillator.

$$U_{ji}(\nu)\exp[-i\Delta\psi^u_{ji}(\nu)] = \int_{-\infty}^\infty C^u_{ji}\exp[-i2\pi\nu\tau]d\tau = FT[C^u_{ji}] \tag{31}$$

There is a similar expression for the lower sideband visibility spectrum. Note that two of the terms in (21-24) are the Hilbert Transforms of (29) and (30). That is, the phase of every frequency component is shifted by $\pi/2$. For example,

$$\int_o^\infty U_{ji}\sin[2\pi\nu\tau + \Delta\psi^u_{ji}]d\tau = -\widehat{C^u_{ji}(\tau)} \tag{32}$$

where the hat denotes the Hilbert transformed quantity. We can now collect these various quantities into the equivalent of (21-24).

$$C_{ji}(\tau) = W^A_{ji}C^u_{ji}(\tau) - W^B_{ji}\widehat{C^u_{ji}(\tau)} + W^A_{ji}C^L_{ji}(\tau) + W^B_{ji}\widehat{C^L_{ji}(\tau)} \tag{33}$$

We can now use the orthogonality of the Walsh functions to separate the upper and lower sidebands. We take the inner product of C_{ji} with each of the terms W^A_{ji} and W^B_{ji} separately.

$$W^A_{ji} * C_{ji}(\tau) = C^u_{ji}(\tau) + C^L_{ji}(\tau) \tag{34}$$

$$W^B_{ji} * C_{ji}(\tau) = -\widehat{C^u_{ji}(\tau)} + \widehat{C^L_{ji}(\tau)} \tag{35}$$

Here is the way in which this last step is realized in the hardware. There are two independent registers in which data are collected, one for W^A_{ji} and one for W^B_{ji}. W^A_{ji} is a three state function: $-2, 0, 2$. It takes on these values at different times during the interval $0 \leq t \leq T$, depending on the structure of the Walsh functions. At any given time in that interval, $C_{ji}(\tau)$ is given whichever of the three weights is appropriate for that moment and stored in the register. This process is continuous, and the data accumulation (integration) is carried out in multiples of T to insure the orthogonality. Accumulation in the W_{ji}^B register happens simultaneously with weighting according to its Walsh functions. After a suitable integration time (e.g., 60 seconds), the power spectra can be constructed from Fourier transforms of the two sets of data.

$$FT[W_{ji}^A * C_{ji}(\tau)] = U_{ji} \exp[i\Delta\psi_{ji}^u] + L_{ji} \exp[-i\Delta\psi_{ji}^L] \tag{36}$$

$$FT[W_{ji}^B * C_{ji}(\tau)] = -H(\nu)U_{ji} \exp[i\Delta\psi_{ji}^u] + H(\nu)L_{ji} \exp[-i\Delta\psi_{ji}^L], \tag{37}$$

where $H(\nu) = -isgn(\nu)$. Finally, we combine these last two quantities to form the output, $O_{ji}(\nu)$, as follows.

$$O_{ji}(\nu) = FT[W_{ji}^A * C_{ji}(\tau)] - iFT[W_{ji}^B * C_{ji}(\tau)] \tag{38}$$

$$= (1 + sgn(\nu))U_{ji} \exp[i\Delta\psi_{ji}^u] + (1 - sgn(\nu))L_{ji} \exp[-i\Delta\psi_{ji}^L] \tag{39}$$

Hence, $O_{ji}(\nu)$ is the upper sideband for $\nu \geq 0$ and the lower sideband for $\nu \leq 0$.

2 The Measurement of Circular Polarization with the System

The measurement of circular polarization is best accomplished by the cross- correlation of two orthogonal linear polarizations. This may be achieved by introducing a half-wave plate in front of the linearly polarized feed of one of the antennas. As a result, the system measures the correlation of two orthogonal linear polarizations. Let them be vertical and horizontal in the usual convention.

(a) If unpolarized radiation is incident, there will be no correlation.

(b) If linear polarization parallel to either of these directions is incident, there will be no correlation.

(c) There will be perfect correlation if there is linear polarization incident which is at forty five degrees to the vertical and horizontal directions. This corresponds to the Stoke's parameter U.

(d) If there is circular polarization incident, there will be perfect correlation but at a phase angle of $\pi/2$. The sign of the correlation will tell what is the sense of the circular polarization. In fact, the quadrature response will be proportional to the difference between right and left circular polarization. As a result of the rapid phase switching of the second LO with the system rapidly comparing right and left circular polarization, no (in)frequent switching of the feeds is necessary.

(e) The summary of the above polarization properties is that the system is sensitive to $U + iV$ with rapid switching included (at no extra charge). Note that the sideband separation continues to work as it does for the normally polarized system.

WJW
5/16/94