BIMA
Technical Memo #77

Error Correction in Quadrature Down-Converters

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Nov. 17 1999

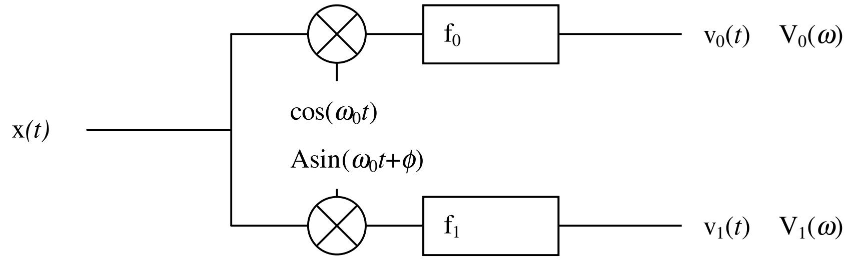
Abstract

The requirements for greater noise immunity from outside sources, such as satellites, in radio astronomy require systems with greater dynamic range so that the interfering signal doesn’t wash out the very quiet signals from deep space. One of the classic methods for bringing a high frequency RF signal down to a lower more manageable IF frequency range is the quadrature down-converter. While quadrature down-converters are relatively simple, they suffer from sideband separation problems. Loud signals in one sideband can leak into the other and the amount of this leakage depends upon how much time and money is spent upon the circuit. Typically one can expect sideband rejection levels of only 30 dB below the interference even with components of reasonably high precision.

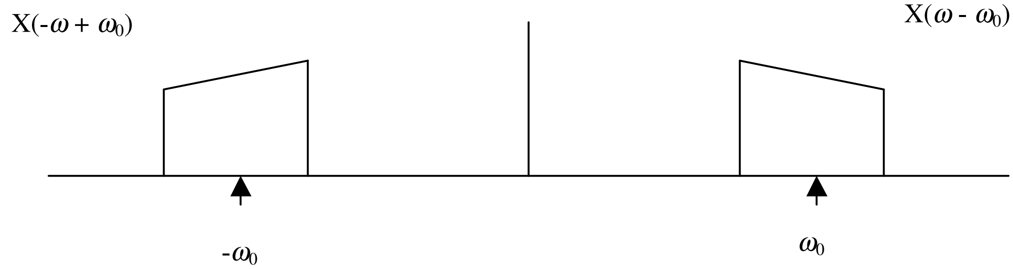
The errors in a quadrature down-converter are easily identified and their effects on the power spectrum can be eliminated, digitally, at the low frequency IF, resulting in a circuit with superior side-band separation. These errors can be identified using the signal being measured so that no separate time consuming calibration is required. As the astronomic measurement is taking place an error function can be determined and used to servo the very slowly changing corrections required as the circuit ages or changes with temperature. As the cost of digital electronics drops it becomes economic to follow a relatively inexpensive quadrature down-converter with digital electronics designed to compensate for its’ short-comings.

Discussion

Quadrature Down-converter



Input Spectrum



Consider a real input signal x(t) with Fourier transform

$$\Im\{x(t)\} = X(-\omega + \omega_0) + X(\omega - \omega_0)$$

It is mixed with two local oscillators that are not quite in perfect quadrature.

$$\Im\{\cos(\omega_0 t)\} = \frac{1}{2}\delta(\omega - \omega_0) + \frac{1}{2}\delta(\omega + \omega_0)$$

$$\Im\{A\sin(\omega_0 t + \phi)\} = \frac{A\cos\phi}{2} j(-\delta(\omega - \omega_0) + \delta(\omega + \omega_0)) + \frac{A\sin\phi}{2}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

The mixer output for the upper branch is then

$$\Im\{x(t)\} \otimes \Im\{\cos(\omega_0 t)\} = \frac{1}{2}X(\omega) + \frac{1}{2}X(-\omega) + \frac{1}{2}X(\omega + 2\omega_0) + \frac{1}{2}X(-\omega - 2\omega_0)$$

Where $\otimes$ denotes convolution.
The filters f_0 and f_1 completely eliminate the components at $2\omega_0$.

Gain Error

It is only necessary to consider differences between the two branches. The filter f_0 is assumed to be ideal with a gain of one and no phase error across the passband. Then the Fourier transform of the output of the upper branch is

$$V_0(\omega) = \frac{1}{2}X(\omega) + \frac{1}{2}X(-\omega)$$

The mixer output for the lower branch is

$$\Im\{x(t)\}\otimes\Im\{A\sin(\omega_0 t+\phi\}=\frac{A\cos(\phi)}{2}j\left(-X(\omega)+X(-\omega)+X(\omega+2\omega_0)-X(-\omega-2\omega_0)\right)$$
$$+\frac{A\sin(\phi)}{2}\left(X(\omega)+X(-\omega)+X(\omega+2\omega_0)+X(-\omega-2\omega_0)\right)$$

The filter f_1 is not ideal and all of the filter errors are lumped into it. The Fourier transform of the output of the lower branch is

$$V_1(\omega)=F_1(\omega)\frac{A\cos(\phi)}{2}j\left(-X(\omega)+X(-\omega)\right)+F_1(\omega)\frac{A\sin(\phi)}{2}\left(X(\omega)+X(-\omega)\right)$$

The power spectra emerging from the two branches may now be determined. First, the upper branch power spectrum is

$$V_0(\omega)V_0^*(\omega)=\frac{1}{4}X(\omega)X^*(\omega)+\frac{1}{4}X(-\omega)X^*(-\omega)+\frac{1}{4}X(\omega)X^*(-\omega)+\frac{1}{4}X(-\omega)X^*(\omega)$$

The $X(\omega)X^*(-\omega)$ and $X(-\omega)X^*(\omega)$ terms represent noise that has the appearance of synchronous demodulation about the LO. These terms will integrate to zero. The power spectrum of the lower branch is

$$V_1(\omega)V_1^*(\omega)=$$
$$F_1(\omega)F_1^*(\omega)\left\{\begin{array}{l}\frac{A^2\cos^2(\phi)}{4}\left[X(\omega)X^*(\omega)+X(-\omega)X^*(-\omega)-X(\omega)X^*(\omega)-X(\omega)X^*(\omega)\right]+\\ \frac{A^2\sin^2(\phi)}{4}\left[X(\omega)X^*(\omega)+X(-\omega)X^*(-\omega)+X(\omega)X^*(\omega)+X(\omega)X^*(\omega)\right]\end{array}\right\}$$

Let $X(\omega)X^*(\omega)+X(-\omega)X^*(-\omega)=R(\omega)$

and $X(\omega)X^*(-\omega)+X(-\omega)X^*(\omega)=AM(\omega)$ then

$$V_0(\omega)V_0^*=\frac{1}{4}\left[R(\omega)+AM(\omega)\right]$$

and

$$V_1(\omega)V_1^*(\omega)=F_1(\omega)F_1^*(\omega)\left\{\frac{A^2\cos^2(\phi)}{4}\left[R(\omega)-AM(\omega)\right]+\frac{A^2\sin^2(\phi)}{4}\left[R(\omega)+AM(\omega)\right]\right\}$$

$$V_1(\omega)V_1^*(\omega)=\frac{F_1(\omega)F_1^*(\omega)A^2}{4}\left\{R(\omega)+AM(\omega)\left[\sin^2(\phi)-\cos^2(\omega)\right]\right\}$$

The gain of the filter f_1 as a function of frequency is

$$g(\omega) = \sqrt{F_1(\omega)F_1^*(\omega)}$$

Neglecting the noise terms, the gain imbalance of the system may be readily determined.

$$Ag(\omega) = \sqrt{\frac{V_1(\omega)V_1^*(\omega)}{V_0(\omega)V_0^*(\omega)}}$$

It remains to solve for the phase correction terms. It is necessary to consider the phase error of the LO and the phase error introduced by the filter pair f_0 and f_1.

<u>LO Phase Error</u>

First determine the cross power spectrum between the two branches.

$$V_0(\omega)V_1^*(\omega) = \frac{AF_1^*(\omega)}{4}\left[X(\omega)+X(-\omega)\right]\left\{j\cos(\phi)\left[X^*(\omega)-X^*(-\omega)\right]+\sin(\phi)\left[X^*(\omega)+X^*(-\omega)\right]\right\}$$

$$V_0(\omega)V_1^*(\omega) = \frac{AF_1^*(\omega)}{4}\begin{Bmatrix} j\cos(\phi)\left[X(\omega)X^*(\omega)-X(-\omega)X^*(-\omega)+X(-\omega)X^*(\omega)-X(\omega)X^*(-\omega)\right] \\ +\sin(\phi)\left[X(\omega)X^*(\omega)+X(-\omega)X^*(-\omega)+X(-\omega)X^*(\omega)+X(\omega)X^*(-\omega)\right]\end{Bmatrix}$$

As before, the $X(\omega)X^*(-\omega)$ and $X(-\omega)X^*(\omega)$ terms represent noise that has the appearance of synchronous demodulation about the LO which will integrate to zero.

Let $X(\omega)X^*(\omega)-X(-\omega)X^*(-\omega)=I(\omega)$

and $X(\omega)X^*(-\omega)-X(-\omega)X^*(\omega)=PM(\omega)$ then

$$V_0(\omega)V_1^*(\omega) = \frac{AF_1^*(\omega)}{4}\left\{j\cos(\phi)\left[I(\omega)+PM(\omega)\right]+\sin(\phi)\left[R(\omega)+AM(\omega)\right]\right\}$$

Since the down-converter is not balanced

$$V_0(\omega)V_1^*(\omega) \neq -V_1(\omega)V_0^*(\omega)$$

$$V_1(\omega)V_0^*(\omega) = \frac{AF_1(\omega)}{4}\left\{j\cos(\phi)\left[-X(\omega)+X(-\omega)\right]+\sin(\phi)\left[X(\omega)+X(-\omega)\right]\right\}\left[X^*(\omega)+X^*(-\omega)\right]$$

$$V_1(\omega)V_0^*(\omega) = \frac{AF_1(\omega)}{4}\begin{Bmatrix} j\cos(\phi)\left[-X(\omega)X^*(\omega)+X(-\omega)X^*(-\omega)+X(-\omega)X^*(\omega)-X(\omega)X^*(-\omega)\right] \\ +\sin(\phi)\left[X(\omega)X^*(\omega)+X(-\omega)X^*(-\omega)+X(-\omega)X^*(\omega)+X(\omega)X^*(-\omega)\right]\end{Bmatrix}$$

$$V_1(\omega)V_0^*(\omega) = \frac{AF_1(\omega)}{4}\{j\cos(\phi)[-I(\omega)+PM(\omega)]+\sin(\phi)[R(\omega)+AM(\omega)]\}$$

Let $F_1(\omega) = F_R(\omega) + jF_I(\omega)$

$$V_0(\omega)V_1^*(\omega) = \frac{A(F_R(\omega) - jF_I(\omega))}{4}\{j\cos(\phi)[I(\omega)+PM(\omega)]+\sin(\phi)[R(\omega)+AM(\omega)]\}$$

$$V_0(\omega)V_1^*(\omega) = \frac{A}{4}F_R(\omega)\{j\cos(\phi)[I(\omega)+PM(\omega)]+\sin(\phi)[R(\omega)+AM(\omega)]\}$$

$$+\frac{A}{4}F_I(\omega)\{\cos(\omega)[I(\omega)+PM(\omega)]-j\sin(\phi)[R(\omega)+AM(\omega)]\}$$

$$V_1(\omega)V_0^*(\omega) = \frac{A}{4}F_R(\omega)\{j\cos(\phi)[-I(\omega)+PM(\omega)]+\sin(\phi)[R(\omega)+AM(\omega)]\}$$

$$+\frac{A}{4}F_I(\omega)\{\cos(\omega)[I(\omega)-PM(\omega)]+j\sin(\phi)[R(\omega)+AM(\omega)]\}$$

Determine the even part of the cross power spectrum.

$$E(\omega) = \tfrac{1}{2}[V_1(\omega)V_0^*(\omega) + V_0(\omega)V_1^*(\omega)]$$

$$E(\omega) = \frac{A}{4}F_R(\omega)\{j\cos(\phi)[PM(\omega)]+\sin(\phi)[R(\omega)+AM(\omega)]\}+\frac{A}{4}F_I(\omega)[\cos(\omega)I(\omega)]$$

The even part, except for a noise term, is pure real. Eliminating the noise terms

$$E_R(\omega) = \frac{A}{4}F_R(\omega)\sin(\phi)R(\omega) + \frac{A}{4}F_I(\omega)\cos(\phi)I(\omega)$$

Determine the odd part.

$$O(\omega) = \tfrac{1}{2}[V_0(\omega)V_1^*(\omega) - V_1(\omega)V_0^*(\omega)]$$

$$O(\omega) = \frac{A}{4}F_R(\omega)\,j\cos(\phi)I(\omega) - \frac{A}{4}F_I(\omega)\{-\cos(\phi)PM(\omega) + j\sin(\phi)[R(\omega)+AM(\omega)]\}$$

Except for a noise term, the odd part is pure imaginary. Eliminating the noise terms

$$O(\omega) = j\frac{A}{4}F_R(\omega)\cos(\phi)I(\omega) - j\frac{A}{4}F_I(\omega)\sin(\omega)R(\omega)$$

or

$$O_I(\omega) = \frac{A}{4} F_R(\omega)\cos(\phi) I(\omega) - \frac{A}{4} F_I(\omega)\sin(\omega) R(\omega)$$

It becomes convenient to put the real and imaginary filter terms into magnitude and phase form.

Let $F_R(\omega) = g(\omega)\cos(\psi(\omega))$ and $F_I(\omega) = g(\omega)\sin(\psi(\omega))$

$$E_R(\omega) = \frac{Ag(\omega)}{4}\cos(\psi(\omega))\sin(\phi)R(\omega) + \frac{Ag(\omega)}{4}\sin(\psi(\omega))\cos(\phi)I(\omega)$$

$$O_I(\omega) = \frac{Ag(\omega)}{4}\cos(\psi(\omega))\cos(\phi)I(\omega) - \frac{Ag(\omega)}{4}\sin(\psi(\omega))\sin(\omega)R(\omega)$$

The above may be corrected for A and g(ω) using the values found previously.

$$E_R(\omega) = \cos(\psi(\omega))\sin(\phi)R(\omega) + \sin(\psi(\omega))\cos(\phi)I(\omega)$$

$$O_I(\omega) = \cos(\psi(\omega))\cos(\phi)I(\omega) - \sin(\psi(\omega))\sin(\omega)R(\omega)$$

$$\begin{bmatrix} E_R(\omega) \\ O_I(\omega) \end{bmatrix} = \begin{bmatrix} \cos\psi(\omega) & \sin\psi(\omega) \\ -\sin\psi(\omega) & \cos\psi(\omega) \end{bmatrix} \begin{bmatrix} \sin(\phi)R(\omega) \\ \cos(\phi)I(\omega) \end{bmatrix}$$

$$\begin{bmatrix} \cos\psi(\omega) & -\sin\psi(\omega) \\ \sin\psi(\omega) & \cos\psi(\omega) \end{bmatrix} \begin{bmatrix} E_R(\omega) \\ O_I(\omega) \end{bmatrix} = \begin{bmatrix} \sin(\phi)R(\omega) \\ \cos(\phi)I(\omega) \end{bmatrix}$$

Solve for the local oscillator phase error.

$$\frac{E_R(\omega)\cos\psi(\omega) - O_I(\omega)\sin\psi(\omega)}{R(\omega)} = \sin(\phi)$$

As $\omega \to 0$ $\psi(\omega) \to 0$ so

$$\underline{\frac{E_R(0)}{R(0)} = \sin(\phi)}$$

The values of $E_R(0)$ and $R(0)$ are often unreliable. $\sin(\phi)$ can be obtained by doing a linear regression on

$$\frac{E_R(\omega)}{R(\omega)} = \sin(\phi)$$

for values of ω close to but not including 0.

Filter Phase Error

All that is now necessary is to solve for the filter phase error $\psi(\omega)$.

$$\frac{E_R(\omega)}{R(\omega)}\cos\psi(\omega)-\frac{O_I(\omega)}{R(\omega)}\sin\psi(\omega)=\sin(\phi)$$

This expression may be solved using a recursive technique. First, define an error function

$$f_{err}(\psi(\omega))=\frac{E_{R(}(\omega)}{R(\omega)}\cos\psi(\omega)-\frac{O_I(\omega)}{R(\omega)}\sin\psi(\omega)-\sin(\phi)$$

Find the rate of change of f_{err} as a function of ψ

$$\frac{df_{err}(\psi(\omega))}{d\psi(\omega)}=-\frac{E_{R(}(\omega)}{R(\omega)}\sin\psi(\omega)-\frac{O_I(\omega)}{R(\omega)}\cos\psi(\omega)$$

Start with $\psi(0)=0$.

Solve for $f_{err}(\psi(\omega_n)$ using the last value obtained for $\psi(\omega_{n-1})$

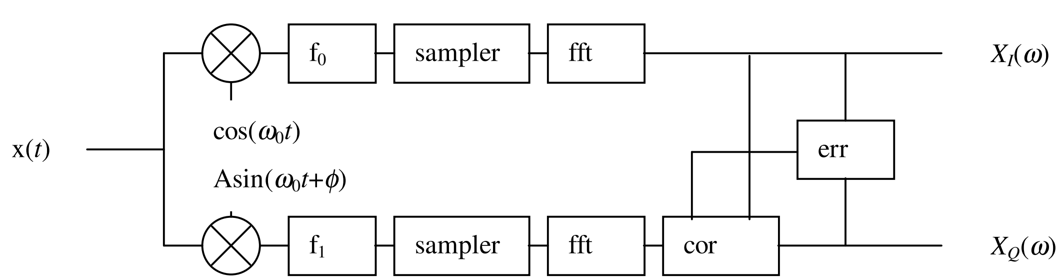
Solve for $\dfrac{df_{err}(\psi(\omega_{n-1}))}{d\psi(\omega_{n-1})}$

Determine the $\Delta\psi$ required to approximate the correct answer.

$$f_{err}(\psi(\omega_n))\times\frac{d\psi(\omega_{n-1})}{df(\psi(\omega_{n-1}))}=\Delta\psi$$

Evaluate $\psi(\omega_{n-1})+\Delta\psi=\psi(\omega_n)$

Quadrature Mixer with Error Correction



Error correction may be done after a power spectrum has been measured by correcting $V_0(\omega)V_0^*(\omega)$, $V_1(\omega)V_1^*(\omega)$, $V_0(\omega)V_1^*(\omega)$ and $V_1(\omega)V_0^*(\omega)$ and assembling the corrected parts into the final spectrum or it may be done as the measurement is taking place. An error detection circuit "err" measures any error in the system and averages those errors over a long period of time. The correction "cor" is applied to the output of an FFT. Filter error is easily corrected by multiplying each of the frequency channels on one of the legs by an amplitude and phase term.

In order to see how to apply the LO correction it is only necessary to expand the output of the LO.

$$x(t)\sin(\omega_0 t+\phi) = x(t)\cos(\phi)\sin(\omega_0 t) + x(t)\sin(\phi)\cos(\omega_0 t)$$

$$x(t)\cos(\phi)\sin(\omega_0 t) = x(t)\sin(\omega_0 t+\phi) - x(t)\sin(\phi)\cos(\omega_0 t)$$

$$x(t)\sin(\omega_0 t) = \frac{x(t)\sin(\omega_0 t+\phi)}{\cos(\phi)} - x(t)\tan(\phi)\cos(\omega_0 t)$$

$$\Im\{x(t)\cos(\omega_0 t)\} = V_0(\omega)$$

LO phase error can be corrected by the vector subtraction of a small fraction of one quadrature from the other after the filter has been corrected. The small amount of scaling required can be accomplished with the amplitude adjustment of the filter error correction.

<u>Error Correction</u>