Bima Memo #49

Correlator Normalization With Power Level Change

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The sampler used in the BIMA correlator quantizes the signal into four levels. This coarse quantization causes the correlation function to be distorted. The distortion may be removed by first normalizing the correlation function and then scaling each measured correlation coefficient by a correction function. The proper function to use and the normalization depends upon the input power level relative to the sampler clipping level. The normalization to use is the topic of this memo.

Prior to correction, the correlation function must be normalized with the zero lag of the autocorrelation function. When an autocorrelation measurement is not available, the zero lag may be determined from the ratio of the clipping level to the square root of the input power level v/σ. Prior to a cross-correlation measurement, the correlator may be configured as an autocorrelator and the zero lag measured as part of a calibration procedure. The zero lag can then be used to determine the v/σ ratio.

An ideal 4 level correlator multiplier would conform to the following table.

	+2	+1	−1	−2
+2	+4	+2	−2	−4
+1	+2	+1	−1	−2
−1	−2	−1	+1	+2
−2	−4	−2	+2	+4

Ideal 2X2 multiplication table
4 level

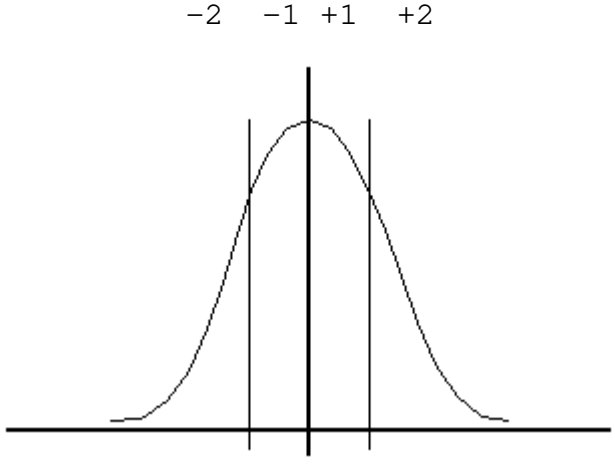
In order to cut down on the amount of logic required, a modified multiplier is used in the BIMA correlator.

	+2	+1	−1	−2
+2	+3	+1	−1	−3
+1	+1	+0	−0	−1
−1	−1	−0	+0	+1
−2	−3	−1	+1	+3

Cooper 1970 method
modified 4 level

The Autocorrelation Case

Let the autocorrelation zero lag count = n
Let the maximum possible count = max
Let the sampler clipping level = v
Let the total power entering the sampler = σ^2



probability distribution of the signal
as it enters the sampler showing
the clipping level v

We are interested in the case of perfect correlation so the only outputs from the zero lag multiplier are either +3 or 0. If v were equal to zero then the multiplier would always produce a 3 for each sample and the count for the total integration would be max. If we assume some finite value for v then

$$\frac{n}{max} = 1-2\int_0^v \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-x^2}{2\sigma^2}}\, dx \quad \text{let } \alpha = \frac{x}{\sigma\sqrt{2}} \quad \sigma\sqrt{2}\,d\alpha = dx \quad \rho = \frac{v}{\sigma}$$

$$\frac{n}{max} = 1-2\int_0^{\frac{\rho}{\sqrt{2}}} \frac{1}{\sqrt{\pi}}\, e^{-\alpha^2}\, d\alpha$$

The error function is of the form

$$\operatorname{erf}\frac{\rho}{\sqrt{2}} = 2\int_0^{\frac{\rho}{\sqrt{2}}} \frac{1}{\sqrt{\pi}}\, e^{-t^2}\, dt$$

then

$$\frac{n}{max} = 1 - \operatorname{erf}\frac{\rho}{\sqrt{2}}$$

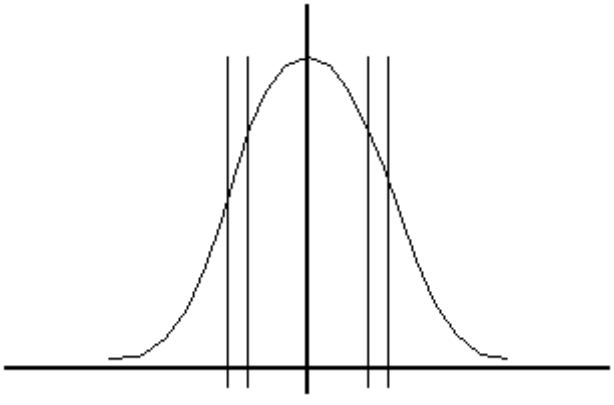
or

$$\frac{v}{\sigma} = \rho = \sqrt{2}\operatorname{erf}^{-1}\left(1-\frac{n}{max}\right)$$

Given the ratio of clipping level to square root of the input power ρ, it is possible to determine the proper normalization; or given the autocorrelator zero lag magnitude (normalization), it is possible to determine the clipping to root power ratio ρ.

The Cross-Correlation Case

The problem becomes more complex for the cross-correlation case because each of the samplers is sampling a different ρ.



probability distribution of the same signal entering samplers with different clipping levels v_1 and v_2

Once again the correlation must be normalized relative to perfect correlation. The probability of getting a 3 out of the multiplier is determined by the sampler with the largest ρ since when the signal voltage is above its clipping level it will be above the clipping level of the other sampler as well.

$$P_3 = 1-2\int_0^{v_2}\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-x^2}{2\sigma^2}}\,dx$$

The sampler with the lowest clipping level will sometimes be producing a +2 when the high ρ sampler is producing a +1 so a product weight of 1 is sometimes possible.

$$P_1 = 1-2\int_0^{v_1}\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-x^2}{2\sigma^2}}\,dx - P_3$$

The output of the multiplier is then proportional to

$$P_1 + 3P_3 = 1-2\int_0^{v_1}\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-x^2}{2\sigma^2}}\,dx +2 -4\int_0^{v_2}\frac{1}{\sigma\sqrt{2\pi}}\,e^{\frac{-x^2}{2\sigma^2}}\,dx$$

or

$$P_1 + 3P_3 = 3 - \mathrm{erf}\frac{\rho_1}{\sqrt{2}} - 2\mathrm{erf}\frac{\rho_2}{\sqrt{2}} \quad \text{where} \quad \rho_2 > \rho_1$$

note

$$P_1 + 3P_3 = 3 \quad \text{when } \rho_1 = \rho_2 = 0$$

which is proportional to max

$$\frac{n}{\max} = \frac{1}{3}P_1 + P_2 = 1 - \frac{1}{3}\mathrm{erf}\frac{\rho_1}{\sqrt{2}} - \frac{2}{3}\mathrm{erf}\frac{\rho 2}{\sqrt{2}}$$

where n is the proper normalization factor to use.
Notice that this leads to a rather interesting result.
since

$$\frac{n_1}{\max} = 1 - \mathrm{erf}\frac{\rho_1}{\sqrt{2}} \quad \text{autocorrelating the first sampler}$$

and

$$\frac{n_2}{\max} = 1 - \mathrm{erf}\frac{\rho_2}{\sqrt{2}} \quad \text{autocorrelating the second sampler}$$

the proper normalization to use in the case of a cross-correlation is

$$n = \frac{2n_2+n_1}{3} \quad \text{where} \quad n_1 > n_2$$

In a linear system the proper normalization to use would be the geometric mean of the two normalization factors.

$$n \neq \sqrt{n_1 n_2}$$

It is very simple to determine the proper normalization to use if the zero lag of the autocorrelation function is known for each of the samplers in a cross-correlation

measurement. If gain fluctuations occur during the cross-correlation measurement then the proper normalization to use is known exactly if the zero lag autocorrelation measurement is determined over the same period of time for each sampler. It is simply

$$n = \frac{2n_2+n_1}{3} \quad \text{where} \quad n_1 > n_2$$

If a known gain change takes place since the last calibration as may occur with a delay line switch then

$$\frac{n_1}{max} = 1 - \text{erf}\frac{\rho_1}{g\sqrt{2}}$$

where g represents the gain change for the first sampler, n_1 is the new normalization for that sampler and

$$n = \frac{2n_2+n_1}{3} \quad \text{where} \quad n_1 > n_2$$

is the correction for the cross-correlator.